

Analysis of Circular Horn Antennas Using the FD-TD Method.

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Abstract

Application of the vector two-dimensional FD-TD method to the analysis of circular horn antennas is presented. An antenna should maintain axial symmetry of boundary conditions and can be arbitrarily shaped in two others directions - in the cylindrical coordinate system. Medium filling the antenna can be inhomogeneous. Modes of excitation of sinusoidal dependence on angle can be considered. A full-wave solution is obtained.

An example of a circular horn antenna has been computed and compared with experimental data.

I. INTRODUCTION

Recently, development of time-domain methods like the finite-difference time-domain (FD-TD) and transmission line matrix (TLM) method proceeds in two directions. The first one is development of the full 3-D analysis. This is the most flexible way of solving the Maxwell equations, but it is very time and memory consuming. Alternative to the 3-D method is the 2-D method. A microwave circuit can be accurately described by a 2-D model if the electromagnetic field within the circuit along one axis is given by a simple analytic function. So far, the 2-D approach has been mainly applied to circuits described by a scalar wave equation [1, 2]. Lately, a class of vector two-dimensional (2-DV) circuits has also been distinguished [3]. The existing 2-D software has been appropriately modified to enable efficient simulation of 2-DV cases [3]. In this contribution we consider a 2-DV model of a circular horn antenna.

A limitation common to most of the reported 2-D algorithms is that they can be used only for shielded structures. To analyse an antenna we need to simulate

radiative boundary conditions. We have chosen the superabsorption approach introduced by Mei and Fang [5, 6] for a one-dimensional case, and adapted it to the 2-DV calculations.

In the paper, we analyse a circular horn antenna excited by the dominant H<sub>11</sub> mode. We compare the results of calculations with experimental data [4].

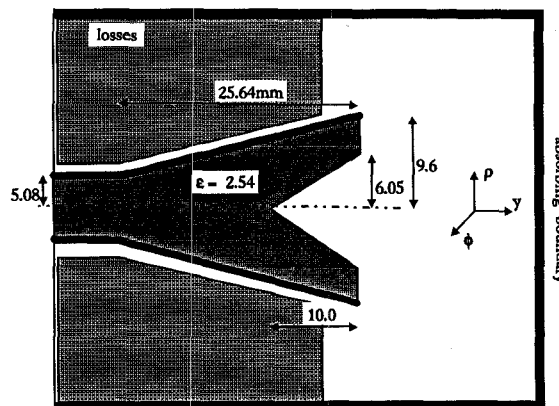


Fig.1. Analyzed horn antenna.

II. 2-DV MODEL

Consider a circular horn antenna presented in Fig.1. We assume that the shape of the antenna and medium parameters are independent of the  $\phi$  coordinate. The input port is excited by the dominant H<sub>11</sub> mode, thus all field components are proportional to  $\sin(\phi)$  or  $\cos(\phi)$ . This kind of a circuit belongs to the 2-DV class distinguished in [3].

Electromagnetic wave propagating in this circuit can be described by two-dimensional vector wave equations written in cylindrical coordinates:

$$\nabla_t \cdot \mathbf{J} = -C \frac{\partial V}{\partial t} \tag{1}$$



$$\nabla_t V - \beta'(\mathbf{i}_\phi \times \mathbf{J}^h) = -L \frac{\partial \mathbf{J}}{\partial t} \quad (2)$$

$$\nabla_t \cdot \mathbf{J}^h = -C^h \frac{\partial V}{\partial t} \quad (3)$$

$$\nabla_t V^h - \beta'(\mathbf{i}_\phi \times \mathbf{J}) = -L^h \frac{\partial \mathbf{J}^h}{\partial t} \quad (4)$$

where:

$$\mathbf{J} = -\mathbf{i}_\phi \times \mathbf{H}_t; \quad V = -\rho E_\phi, \quad (5)$$

$$\mathbf{J}^h = \mathbf{i}_\phi \times \mathbf{E}_t; \quad V^h = -\rho H_\phi, \quad (6)$$

( $\mathbf{H}_\phi$  and  $\mathbf{E}_\phi$  are tangential to the plane  $\phi = \text{const}$ ),

$\nabla_t$  is a two-dimensional operator in ( $\rho, y$ ) coordinates,

$\mathbf{i}_\phi$  is unit vector along  $\phi$  axis,

$\beta' = 1$  (for the dominant  $H_{11}$  mode),

$$C = \varepsilon/\rho; \quad C^h = \mu$$

$$L = \mu/\rho; \quad L^h = \varepsilon/\rho$$

The equations (1-4) describe propagation of two coupled modes =: one - with the angular component  $H_\phi = 0$  (eq. 1-2) and second - with  $E_\phi = 0$  (eq. 3-4). Following notions introduced in [1], we call these two modes  $E_1$  and  $H_1$ , respectively.

The advantage of introducing voltage ( $V, V^h$ ) and current ( $J, J^h$ ) potentials (5-6) to wave equations (1-4) consist in providing simple physical interpretation of our analysis. The corresponding electrical network model is shown in Fig.2.

### III. OPEN BOUNDARY CONDITION

The FD-TD method is easily applicable to the analysis of shielded structures. A problem occurs when we want to simulate the open space. Two general approaches exist. The global approach consists in simulating wave propagation in a vast area around the radiating element. This is accurate but inefficient method. The local approach is based on additional assumptions concerning the wave and leads to relatively simple and efficient absorbing algorithms. We

have applied in our case the superabsorption method proposed by Mei and Fang in [5,6].

It is assumed that in the direction  $n$ , normal to the absorbing surface, an electromagnetic wave is proportional to:

$$V \sim e^{i(\omega t \pm nr)} \quad (7)$$

The simplest absorbing formula results from linear extrapolation of (7) beyond the absorbing boundary. Second order formula for the voltage  $V$  has the following form:

$$V_M^n = V_{M-1}^{n-1} + \frac{1-\rho}{1+\rho} \left[ V_M^{n-1} - V_{M-1}^n \right] \quad (8)$$

where  $V_M^n$  denotes voltage in mesh  $M$  in the time step  $n$ ,

$$\rho = \frac{c\Delta x}{\Delta t}$$

is velocity of the FD-TD algorithm.

By using the formula (8) a certain error is introduced. This error transfers in a leap-frog algorithm to the calculation of current ( $J_1 = J_{M-1}^{n+1/2} + er1$ ). Mei and Fang [5,6] found out that error  $er2$  appearing when the (8) is used directly for current computation ( $J_2 = J_{M-1}^{n+1/2} + er2$ ) has a sign opposite to  $er1$ . Taking an average value of  $J_1$  and  $J_2$  it is possible to drastically decrease the amplitude of a wave reflected from the absorbing boundary.

This procedure was used for simulation of open boundary condition. In our case the wave radiated from the antenna propagates approximately in the radial direction (Fig.3). In the distance of several wavelengths from the antenna (7) is approximately satisfied.

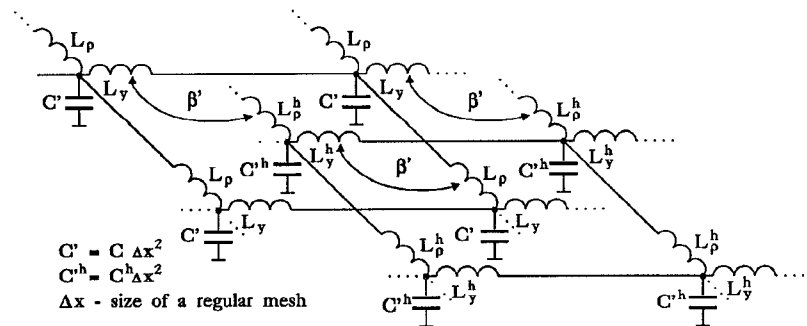


Fig.2. A model of a 2-DV circuit composed of two coupled 2-D models.

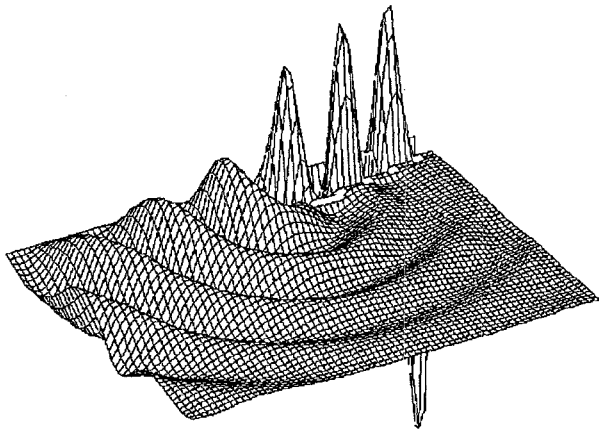


Fig.3. Voltage ( $V = -\rho E_\phi$ ) distribution of a wave radiated from the antenna.

Mei and Fang derived their superabsorbing procedure for the one-dimensional wave propagation. They pointed to possibility of its application to scalar 2-D circuits. In a 2-DV circuit the absorbing condition must be applied to two grids of meshes (two coupled modes). In Fig.4 a fragment of grids of the two modes (associated with potential  $V$  and  $V^h$ ) is shown. We use the superabsorption procedure twice: for calculation of voltage  $V$  and for calculation of currents  $J_y^h$  (or  $J_\rho^h$ , depending on the orientation of the absorbing boundary).

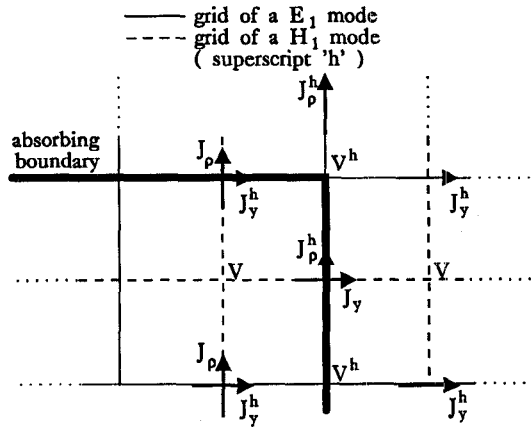


Fig.4. A fragment of a grid of meshes near the corner of absorbing edge.

#### IV. CALCULATIONS

The considered antenna is manufactured in INWATE technology. A polystyrene form is covered by a thin silver

film and then by copper. Dimensions of the antenna are shown in Fig.1 and are optimized for 11.45-12.75 GHz bandwidth [3]. The taper situated at the output performs a matching function.

Efficiency of our algorithm requires that the input port of the circuit be excited by the pure dominant  $H_{11}$  mode. This is achieved by putting Bessel distribution of the amplitude of electromagnetic field (voltage) along the port.

A result of the analysis is the propagation characteristic of the antenna. We have used procedures existing in our 2-D software package. Thus, we have applied pulse excitation and calculated the Fourier transform at a set of output meshes (at a selected frequency). The output meshes have been approximately equidistant, but positioned in various directions from the output of the antenna. Consequently, the Fourier transform directly determines the propagation characteristic.

In the back part of the FD-TD area losses are introduced to ensure attenuation of waves propagating in this direction (Fig.1).

To demonstrate the validity of the method we have calculated the propagation characteristic of the antenna of Fig.1. Comparison of the results of our computation with the experiment (Fig.2.) shows a very good agreement.

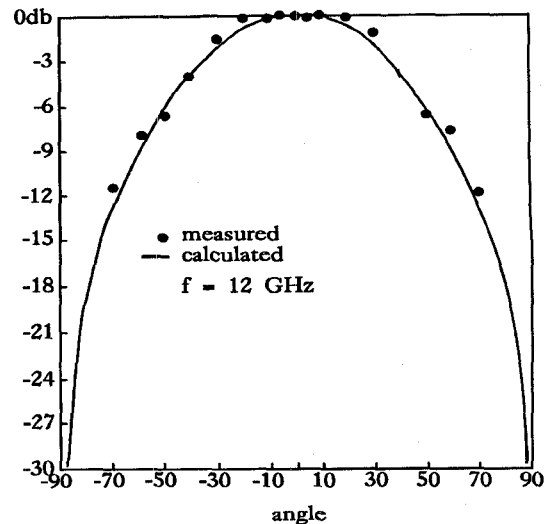


Fig.5. Characteristic of propagation of the antenna.

## V. CONCLUSIONS

An application of the vector two-dimensional FD-TD method to the analysis of a circular horn antenna has been presented. Superabsorption procedure is used for simulation of the open space. Calculated results are in a very good agreement with experimental data.

Significant advantage of the presented method is the possibility of analysing antennas arbitrarily shaped in two dimensions (in this case in the cylindrical coordinate system). Introduction of so called modified meshes allows for good approximation of the shape of a circuit, in spite of relatively rough gridding in the FD-TD algorithm. Presented method gives a full-wave solution of the problem.

## REFERENCES

- [1] W.K.Gwarek, "Analysis of Arbitrarily-Shaped Two-Dimensional Microwave Circuits by Finite-Difference Time-Domain Method," IEEE Trans. on Microwave Theory Tech., vol.MTT-36, pp.738-744, April 1988.
- [2] W.J.R.Hoefer, "The Transmission-Line Matrix Method - Theory and Applications", IEEE Trans. on Microwave Theory Tech., vol.MTT-33, pp.882-892, October 1985.
- [3] C.Mroczkowski, W.Gwarek - "Microwave Circuits Described by Two-Dimensional Vector Wave Equations and Their Analysis by FD-TD Method", European Microwave Conference, Stuttgart, Sept. 1991, vol.1, pp.866-871.
- [4] J.H.Hinken, J.Modelski, F.Henze, U.Klein, "Entwicklung eines DFS-empfangskonverters", Report No. 01 YH 8608, Technische Universitaet Braunschweig, 1988.
- [5] K.K.Mei, J.Fang, "Superabsorption: A Method to Improve Local Absorbing Boundary Conditions" - submitted to J. Comput. Phys.
- [6] X.Zhang, K.K.Mei, "Time-Domain Finite Difference Approach to the Calculation of the Frequency-Dependent Characteristics of Microstrip Discontinuities", IEEE Trans. on Microwave Theory Tech., vol.MTT-36, pp.1775-1778, December 1988.